



Attempt all questions before checking the answer key. Assume fair devices and uniform selection unless stated. Give exact probabilities and identify conditioning events.

A. Very short answers - 1 mark each

1. Define conditional probability $P(A|B)$, including its required condition.
2. State the multiplication theorem of probability.
3. What condition makes two events independent?
4. Can two events of positive probability be both independent and mutually exclusive?
5. State Bayes' theorem for two exhaustive, mutually exclusive causes.

B. Short answers / practice - 2 marks each

6. If $P(A \cap B) = 0.18$ and $P(B) = 0.6$, find $P(A|B)$.
7. If A and B are independent with probabilities 0.4 and 0.5, find $P(A \cap B)$ and $P(A \cup B)$.
8. A card drawn from a standard deck is known to be a face card. Find the probability that it is a king.
9. Two cards are drawn without replacement. Find the probability that both are aces.
10. If $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cup B) = 0.8$, test whether A and B are independent.
11. If $P(A \cap B) = 0.12$ and $P(B) = 0.4$, find $P(A|B)$.
12. Independent A, B have probabilities 0.2, 0.3. Find $P(\text{neither})$.
13. A card is red. Find probability that it is a heart.
14. Two fair coins are tossed. Given at least one head, find probability of two heads.
15. Are independent events necessarily mutually exclusive?
16. A bag has 3 red, 2 blue balls. Find probability of two reds without replacement.
17. For independent events $P(A) = 1/2$, $P(B) = 1/4$, find union probability.
18. Can $P(A|B)$ exceed 1 when $P(B) > 0$?
19. Two identical boxes contain respectively 2 red/1 blue and 1 red/2 blue. Pick a box randomly then a red ball. Find probability it was the first box.
20. Find $P(B|A)$ if $P(A) = 0.5$ and $P(A \cap B) = 0.2$.

C. Short answers - 3 marks each

21. A bag contains 4 red and 6 blue balls. Two are drawn without replacement. Find the probability that the second is red given that the first is blue.

22. Two dice are thrown. Given that their sum is 8, find the probability that one die shows 3.

23. Prove that if A and B are independent, A' and B are also independent.

24. Box I contains 3 red and 2 blue balls; Box II contains 1 red and 4 blue balls. A box is chosen at random. Find the probability of drawing red.

25. A test detects a condition in 90% of affected people and gives a false positive in 5% of unaffected people. If prevalence is 10%, find the probability of a positive test.

D. Long answers - 5 marks each

26. State and derive Bayes' theorem for a finite partition. Explain the roles of prior probability, likelihood and posterior probability using a two-box selection example.
27. Machines A, B and C make 20%, 30% and 50% of a factory's output, with defect rates 1%, 2% and 3%. Find the probability an item is defective and the probability it came from C given that it is defective.
28. An urn is chosen at random from three urns containing respectively (2 red, 3 blue), (4 red, 1 blue) and (3 red, 2 blue). A red ball is drawn. Find the probability of each urn and identify the most likely source.
29. In a population, 2% have a condition. A test has sensitivity 95% and specificity 90%. Find the probability of a positive result and the probability of the condition given a positive result. Explain why these probabilities differ.
30. Three students independently solve a problem with probabilities $1/2$, $2/3$ and $3/4$. Find probabilities that none solves it, at least one solves it and exactly two solve it. Show the event decomposition.

ANSWER KEY - Numerical / short answers

- 4) No. 6) 0.3. 7) 0.2; 0.7. 8) $1/3$. 9) $1/221$. 10) Intersection $0.3 = 0.6 \times 0.5$; independent. 11) 0.3. 12) 0.56. 13) $1/2$. 14) $1/3$. 15) No. 16) $3/10$. 17) $5/8$. 18) No. 19) $2/3$. 20) 0.4. 21) $4/9$. 22) $2/5$. 24) $2/5$. 25) 0.135. 27) $P(\text{defective}) = 0.023$; $P(C|\text{defective}) = 15/23$. 28) $2/9, 4/9, 1/3$; second urn most likely. 29) $P(+)=0.117$; $P(\text{condition}|+)=19/117 \approx 0.1624$. 30) None $1/24$; at least one $23/24$; exactly two $11/24$.