



Attempt all questions before checking the answer key. State domains where division is used. Write general solutions with arbitrary constants unless initial data are given.

A. Very short answers - 1 mark each

1. Define the order of a differential equation.
2. Find order and degree of $(y'')^2 + y' + y = 0$.
3. Is the degree of $y' + \sin(y') = x$ defined? Explain.
4. What distinguishes a particular solution from a general solution?
5. State the integrating factor for $y' + P(x)y = Q(x)$.

B. Short answers / practice - 2 marks each

6. Solve $dy/dx = 3x^2$.
7. Solve $dy/dx = 2y$, given $y(0) = 3$.
8. Solve $dy/dx = x/y$, retaining the constant of integration.
9. Find the integrating factor for $dy/dx + (2/x)y = x$, $x > 0$.
10. Verify that $y = Ce^{(-x)} + x - 1$ solves $y' + y = x$.
11. Find order of $y''' + y = 0$.
12. Solve $y' = 5$.
13. Solve $y' = y$ with $y(0) = 2$.
14. Find integrating factor for $y' + 3y = x$.
15. Solve $y' = x^2/y$.
16. Is $y = 0$ a solution of $y' = 2y$?
17. Solve $y' + y = 0$.
18. Find degree of $(y'')^3 + y' = 0$.
19. Solve $y' = \cos x$ with $y(0) = 1$.
20. Solve $y' - 2y = 0$, $y(0) = 4$.

C. Short answers - 3 marks each

21. Solve $dy/dx = (1 + x^2)/(1 + y^2)$ by separation.

22. Solve $dy/dx + y = e^x$.

23. Solve $x dy/dx - y = x^2$ for $x > 0$.

24. Solve $dy/dx = (x + y)/x$ using $y = vx$, with $x > 0$.

25. Solve $dx/dy + x = e^{(2y)}$.

D. Long answers - 5 marks each

26. Solve $dy/dx + y \tan x = \sin x$ on $-\pi/2 < x < \pi/2$, given $y(0) = 2$. Show the integrating-factor steps.

27. Solve $dy/dx = (x + y)/(x - y)$ by homogeneous substitution, stating restrictions and obtaining an implicit solution.

28. Solve $(1 + x^2)dy/dx + 2xy = 1$, given $y(0) = 0$. Verify your particular solution.

29. A population grows at a rate proportional to its size and doubles in 5 years. Form and solve the differential equation and find the time to become eight times its initial size.

30. A substance decays at a rate proportional to the amount present; 80% remains after 2 hours. Derive its amount-time relation and find the time at which 64% remains.

ANSWER KEY - Numerical / short answers

2) Order 2, degree 2. 3) No; it is not polynomial in derivatives. 5) $e^{\int P dx}$. 6) $y = x^3 + C$. 7) $y = 3e^{(2x)}$. 8) $y^2 = x^2 + C$, on intervals with $y \neq 0$. 9) x^2 . 10) Substitution gives $y' + y = x$. 11) 3. 12) $y = 5x + C$. 13) $y = 2e^x$. 14) $e^{(3x)}$. 15) $y^2 = 2x^3/3 + C$, $y \neq 0$ locally. 16) Yes. 17) $y = Ce^{(-x)}$. 18) 3. 19) $y = \sin x + 1$. 20) $y = 4e^{(2x)}$. 21) $y + y^3/3 = x + x^3/3 + C$. 22) $y = e^x/2 + Ce^{(-x)}$. 23) $y = x^2 + Cx$. 24) $y = x(\ln x + C)$. 25) $x = e^{(2y)/3} + Ce^{(-y)}$. 26) $y = \cos x [2 - \ln(\cos x)]$. 27) $(1/2)\ln(x^2 + y^2) - \tan^{-1}(y/x) = C$ locally, $x \neq 0, x \neq y$; branches must be chosen consistently. 28) $y = x/(1 + x^2)$. 29) $P = P_0 2^{(t/5)}$; 15 years. 30) $A = A_0(0.8)^{(t/2)}$; 4 hours.