



Attempt all questions before checking the answer key. Draw the region, state integration limits and give geometric areas as non-negative quantities.

A. Very short answers - 1 mark each

1. What does $\int [a,b] f(x) dx$ represent when $f(x) \geq 0$ on $[a,b]$?

2. Why can a definite integral differ from the total geometric area?

3. Find the area under $y = 2$ from $x = 0$ to $x = 3$.

4. Write an integral for the first-quadrant area inside $x^2 + y^2 = 9$.

B. Short answers / practice - 2 marks each

5. Find the area under $y = x$ between $x = 0$ and $x = 4$.

6. Find the area under $y = x^2$ between $x = 0$ and $x = 2$.

7. Find the area between $y = 4 - x$, the x-axis and the y-axis.

8. Find the area between $y = -x$, $x = 0$, $x = 2$ and the x-axis.

9. Find area under $y=3x$ from 0 to 2.

10. Find area under $y=x^2$ from 1 to 2.

11. Find area under $y=\sin x$ on $[0,\pi]$.

12. Find area between $y=-2$ and x-axis for $0 \leq x \leq 3$.

13. Find first-quadrant area of $x^2+y^2=16$.

14. Can geometric area be negative?

15. Find area under $y=4-x^2$ from $x=0$ to 2.

16. Find area under $y=1/x$ from 1 to e.

17. Find area between $y=\cos x$ and x-axis on $[0,\pi/2]$.

18. Find area bounded by $y^2=4x$ and $x=1$.

19. Find first-quadrant area of $x^2/25+y^2/4=1$.

20. Find area under $y=\sqrt{x}$ for $0 \leq x \leq 4$.

21. Is $\int [-1,1] x dx$ the total area between $y=x$ and the axis?

22. Find area of circle $x^2+y^2=9$ by integrating over four quadrants.

C. Short answers - 3 marks each

23. Find the area under $y = \sqrt{4 - x^2}$ from $x = 0$ to $x = 2$ and verify it geometrically.

24. Find the area bounded by $y^2 = 4x$, the y-axis and $y = 0$, $y = 4$.

25. Find the area between $y = x^2 - 4$ and the x-axis from $x = 0$ to $x = 3$, accounting for the sign change.

26. Find the area inside $x^2/16 + y^2/9 = 1$ in the first quadrant by integration.

D. Long answers - 5 marks each

27. Derive the area πab of the ellipse $x^2/a^2 + y^2/b^2 = 1$ by integration and symmetry. Deduce the circle-area formula as a special case.

28. Find the area of the circle $x^2 + y^2 = 16$ lying to the right of $x = 2$ using definite integrals. Show the limits and a suitable substitution.

29. Find the area enclosed by the parabola $y^2 = 8x$ and the vertical line $x = 2$. Draw the region and explain the symmetry used.

30. Using integration, find the area of the triangle bounded by $y = x$, $y = 0$ and $x = 6$. Then find the vertical line $x = k$ that divides it into two equal areas.

ANSWER KEY - Numerical / short answers

3) 6 square units. 4) $\int [0,3] \sqrt{9-x^2} dx$. 5) 8. 6) $8/3$. 7) 8. 8) 2. 9) 6. 10) $7/3$. 11) 2. 12) 6. 13) 4π . 14) No. 15) $16/3$. 16) 1. 17) 1. 18) $8/3$. 19) $5\pi/2$. 20) $16/3$. 21) No; signed integral 0, total area 1. 22) 9π . 23) π . 24) $16/3$. 25) $23/3$. 26) 3π . 28) $16\pi/3 - 4\sqrt{3}$. 29) $32/3$. 30) 18; $k=3\sqrt{2}$.