



Attempt all questions before checking the answer key. Include the constant of integration for indefinite integrals. The notation $\int [a,b]$ means limits a to b .

A. Very short answers - 1 mark each

1. Evaluate $\int x^3 dx$.
2. Evaluate $\int \cos x dx$.
3. Evaluate $\int 1/x dx$ for $x \neq 0$.
4. Evaluate $\int [0,1] 2x dx$.
5. State the relation between $\int [a,b] f(x) dx$ and $\int [b,a] f(x) dx$.

B. Short answers / practice - 2 marks each

6. Evaluate $\int (3x^2 - 4x + 5) dx$.
7. Evaluate $\int 2x/(x^2 + 1) dx$.
8. Evaluate $\int dx/(1 + x^2)$.
9. Evaluate $\int \sin x \cos x dx$.
10. Evaluate $\int [0,\pi/2] \sin x dx$ and interpret its sign.
11. Evaluate $\int e^{(2x)} dx$.
12. Evaluate $\int \sec^2 x dx$.
13. Evaluate $\int [0,2] x dx$.
14. Evaluate $\int 1/\sqrt{1-x^2} dx$.
15. Evaluate $\int (2x+1)^3 dx$.
16. Evaluate $\int [-1,1] x^3 dx$.
17. Evaluate $\int x/(x^2+4) dx$.
18. Evaluate $\int \ln x dx$, $x > 0$.
19. Evaluate $\int [0,\pi] \sin x dx$.
20. Does every antiderivative of the same continuous function differ only by a constant on an interval?

C. Short answers - 3 marks each

21. Evaluate $\int x e^x dx$ by parts.
22. Evaluate $\int dx/[x(x+1)(x+2)]$ using partial fractions.

23. Evaluate $\int dx/(x^2 + 4x + 5)$.

24. Evaluate $\int [0,\pi/2] \sin^2 x dx$ using a trigonometric identity.

25. Evaluate $\int (3x + 2)/\sqrt{x^2 + 4x + 8} dx$ by splitting the numerator appropriately.

D. Long answers - 5 marks each

26. Evaluate $\int x^2 e^x dx$ by repeated integration by parts and verify the result by differentiation.
27. Evaluate $\int dx/[x(x+1)(x+2)]$ using partial fractions. State intervals on which the antiderivative applies.
28. Using symmetry of definite integrals, evaluate $\int [0,\pi/2] \sin x/(\sin x + \cos x) dx$. State and justify the property used.
29. Evaluate $\int \sqrt{9 - x^2} dx$ using $x = 3 \sin \theta$ and hence find $\int [0,3] \sqrt{9 - x^2} dx$.
30. Evaluate $\int [0,2] |x^2 - 1| dx$ by splitting at the sign change. Compare it with $\int [0,2] (x^2 - 1) dx$ and explain the difference.

ANSWER KEY - Numerical / short answers

- 1) $x^4/4 + C$. 2) $\sin x + C$. 3) $\ln|x| + C$. 4) 1. 5) They are negatives of each other. 6) $x^3 - 2x^2 + 5x + C$. 7) $\ln(x^2 + 1) + C$. 8) $\tan^{-1} x + C$. 9) $\sin^2 x/2 + C$. 10) 1; integrand non-negative. 11) $e^{(2x)}/2 + C$. 12) $\tan x + C$. 13) 2. 14) $\sin^{-1} x + C$, $|x| < 1$. 15) $(2x+1)^4/8 + C$. 16) 0. 17) $(1/2)\ln(x^2+4) + C$. 18) $x \ln x - x + C$. 19) 2. 20) Yes. 21) $e^x(x-1) + C$. 22) $\ln|(x+1)/(x+2)| + C$. 23) $\tan^{-1}(x+2) + C$. 24) $\pi/4$. 25) $3\sqrt{x^2+4x+8} - 4\ln|x+2+\sqrt{x^2+4x+8}| + C$. 26) $e^x(x^2-2x+2) + C$. 27) $(1/2)\ln|x| - \ln|x+1| + (1/2)\ln|x+2| + C$; intervals excluding $-2, -1, 0$. 28) $\pi/4$. 29) $(x/2)\sqrt{9-x^2} + (9/2)\sin^{-1}(x/3) + C$; definite value $9\pi/4$. 30) Area integral 2; signed integral $2/3$.