



Attempt all questions before checking the answer key. Use first- or second-derivative tests and check endpoints in closed-interval optimisation problems.

### A. Very short answers - 1 mark each

1. If  $f'(x) > 0$  throughout an interval, what does this imply about  $f$  there?
2. What additional information is needed before declaring a point with  $f'(a) = 0$  a maximum or minimum?
3. Find the rate of change of circle area with respect to radius.
4. Find critical points of  $f(x) = x^2 - 6x + 5$ .
5. Does  $f(x) = x^3$  have a local maximum or minimum at 0? Justify.

### B. Short answers / practice - 2 marks each

6. Find intervals where  $f(x) = x^2 - 4x$  is increasing or decreasing.
7. Find local extrema of  $f(x) = x^3 - 3x$ .
8. A sphere's radius increases at 0.2 cm/s. Find the rate of volume increase when its radius is 5 cm.
9. Find the maximum and minimum of  $x^2 - 2x + 3$  on  $[0, 3]$ .
10. Show that  $f(x) = x + 1/x$  is decreasing on  $(0, 1)$  and increasing on  $(1, \infty)$ .
11. Find minimum of  $x^2 + 4x + 7$ .
12. Find stationary points of  $x^3 - 12x$ .
13. A circle's radius grows at 1 cm/s. Find area growth at  $r = 3$  cm.
14. Is  $e^x$  increasing on  $\mathbb{R}$ ?
15. Find maximum of  $9 - x^2$  on  $[-2, 2]$ .
16. Find intervals of increase for  $x^3 + 3x$ .
17. Find minimum of  $x + 4/x$  for  $x > 0$ .
18. Find local extrema of  $x^4$ .
19. A cube's side grows at 2 cm/s. Find volume growth at side 3 cm.
20. Explain why a zero first derivative alone is insufficient to establish an extremum.

### C. Short answers - 3 marks each

21. Find intervals of increase/decrease and local extrema of  $x^3 - 6x^2 + 9x + 1$ .

22. A ladder 10 m long rests against a wall. Its foot moves away at 0.5 m/s. Find the speed at which its top descends when the foot is 6 m from the wall.

23. Find the dimensions of the rectangle of maximum area with perimeter 40 m.

24. Find the positive number that minimises  $x + 25/x$  and prove it is a minimum.

25. A cube's volume increases at  $24 \text{ cm}^3/\text{s}$ . Find the rate of increase of its surface area when its side is 2 cm.

### D. Long answers - 5 marks each

26. An open-top box is made by cutting equal squares from corners of a 30 cm square sheet and folding up the sides. Find the cut size giving maximum volume and the maximum volume.

27. Find the dimensions of a closed right circular cylinder of volume  $250\pi \text{ cm}^3$  having minimum total surface area. Prove that the critical point gives the minimum.

28. A farmer has 120 m fencing for three sides of a rectangular plot beside a straight river. Find the dimensions giving maximum area and the maximum area.

29. Find the absolute maximum and minimum of  $f(x) = x^3 - 3x^2 - 9x + 5$  on  $[-2, 5]$ . Show critical-point and endpoint comparisons.

30. A wire 36 cm long is cut into two pieces, one forming a square and the other a circle. Find the cut that minimises the sum of their areas and justify the minimum.

### ANSWER KEY - Numerical / short answers

- 3)  $2\pi$ . 4)  $x = 3$ . 5) No; it increases through zero. 6) Decreasing  $(-\infty, 2)$ , increasing  $(2, \infty)$ . 7) Local max 2 at  $-1$ ; local min  $-2$  at 1. 8)  $20\pi \text{ cm}^3/\text{s}$ . 9) Min 2 at 1; max 6 at 3. 10) 3 at  $x = -2$ . 11)  $x = \pm 2$ . 12)  $6\pi \text{ cm}^2/\text{s}$ . 13) Yes. 14) 9 at  $x = 0$ . 15) All  $\mathbb{R}$ . 16) 4 at  $x = 2$ . 17) Local and absolute minimum 0 at  $x = 0$ ; no local maximum. 18)  $54 \text{ cm}^3/\text{s}$ . 19) Increasing  $(-\infty, 1)$ ,  $(3, \infty)$ ; decreasing  $(1, 3)$ ; max 5 at 1, min 1 at 3. 20)  $0.375 \text{ m/s}$  downward. 21) 10 m  $\times$  10 m. 22)  $x = 5$ ; minimum 10. 23)  $48 \text{ cm}^2/\text{s}$ . 24) Cut 5 cm; maximum  $2000 \text{ cm}^3$ . 25)  $r = 5 \text{ cm}$ ,  $h = 10 \text{ cm}$ . 26) 30 m perpendicular to river, 60 m parallel; area  $1800 \text{ m}^2$ . 27) Maximum 16 at  $x = -1$ ; minimum  $-22$  at  $x = 3$ . 28) Square perimeter  $144/(4 + \pi) \text{ cm}$ ; circle circumference  $36\pi/(4 + \pi) \text{ cm}$ .