



Attempt all questions before checking the answer key. Use radians. Interpret $\ln$ as the natural logarithm and state domains of logarithmic expressions.

A. Very short answers - 1 mark each

1. State the condition for continuity of f at $x = a$.
2. Is every differentiable function continuous? State whether the converse holds.
3. Differentiate e^x .
4. Differentiate $\ln x$ for $x > 0$.
5. Find $d/dx(\tan^{-1}x)$.

B. Short answers / practice - 2 marks each

6. Find k for continuity at zero if $f(x) = \sin x/x$ for $x \neq 0$ and $f(0) = k$.
7. Differentiate $\sin(x^2)$.
8. Differentiate $e^{(3x)} \ln x$ for $x > 0$.
9. If $x^2 + y^2 = 25$, find dy/dx by implicit differentiation.
10. Find the derivative of $\sin^{-1}(2x)$ and state where it exists.
11. Differentiate $\ln(x^2+1)$.
12. Differentiate $\cos 3x$.
13. Find derivative of $e^{(x^2)}$.
14. Is $|x-2|$ differentiable at $x=2$?
15. Find k for $f(x)=kx$ ($x<1$), $f(x)=3$ ($x\geq 1$) to be continuous at 1.
16. Differentiate $x^2 \ln x$ for $x>0$.
17. Find y' if $xy=1$.
18. Differentiate $\tan^{-1}(2x)$.
19. Find second derivative of x^3 .
20. Can a function be continuous but not differentiable at a point?

C. Short answers - 3 marks each

21. Find a and b so that $f(x) = ax + b$ for $x < 1$ and $f(x) = x^2 + 2$ for $x \geq 1$ is both continuous and differentiable at 1.

22. Differentiate $y = x^x$ for $x > 0$ by logarithmic differentiation.

23. If $x = a \cos t$ and $y = a \sin t$, find dy/dx and d^2y/dx^2 where defined.

24. Find dy/dx for $y = \ln[(1+x)/(1-x)]$, $-1 < x < 1$.

25. Show that $|x|$ is continuous but not differentiable at 0 using one-sided limits and derivatives.

D. Long answers - 5 marks each

26. Examine continuity and differentiability of $f(x) = |x - 1| + |x + 1|$ at $x = -1$ and 1. Write its piecewise form and sketch the graph.

27. If $y = (\sin x)^x$ on an interval where $\sin x > 0$, obtain dy/dx by logarithmic differentiation and then compute the derivative at $x = \pi/2$.

28. For $x = a(t - \sin t)$, $y = a(1 - \cos t)$, derive dy/dx and d^2y/dx^2 at regular points. State which parameter values must be excluded.

29. If $y = \sin^{-1}x$, prove $(1 - x^2)y'' - xy' = 0$ for $-1 < x < 1$, giving each differentiation step.

30. Determine constants a and b so that $f(x) = ax^2 + bx + 1$ for $x \leq 1$ and $f(x) = 3x + 2$ for $x > 1$ is continuous and differentiable at 1. Find its derivative on both sides.

ANSWER KEY - Numerical / short answers

2) Yes; converse is false, e.g. $|x|$ at zero. 3) e^x . 4) $1/x$. 5) $1/(1+x^2)$. 6) 1. 7) $2x \cos(x^2)$. 8) $e^{(3x)}(3 \ln x + 1/x)$. 9) $-x/y$ for $y \neq 0$. 10) $2/\sqrt{1-4x^2}$, $|x| < 1/2$. 11) $2x/(x^2+1)$. 12) $-3 \sin 3x$. 13) $2xe^{(x^2)}$. 14) No. 15) $k=3$. 16) $2x \ln x + x$. 17) $-1/x^2$, $x \neq 0$. 18) $2/(1+4x^2)$. 19) $6x$. 20) Yes; $|x|$ at zero. 21) $a=2, b=1$. 22) $x^x(\ln x + 1)$. 23) $-\cot t$; $-1/(a \sin^3 t)$, with $a \neq 0$, $\sin t \neq 0$. 24) $2/(1-x^2)$. 25) Continuous at both, differentiable at neither; $-2x$ for $x < -1$, 2 for $-1 \leq x \leq 1$, $2x$ for $x > 1$. 26) $y' = (\sin x)^x [\ln(\sin x) + x \cot x]$; value 0. 27) $dy/dx = \cot(t/2)$; $d^2y/dx^2 = -1/[a(1 - \cos t)^2]$; exclude $t = 2n\pi$, $a=0$. 28) $a=-1, b=5$; left derivative $-2x+5$; right derivative 3.