



Attempt all questions before checking the answer key. Matrix notation $[a, b; c, d]$ lists successive rows. Use I for the identity of appropriate order.

A. Very short answers - 1 mark each

1. State the order of a matrix with 3 rows and 2 columns.
2. When are two matrices equal?
3. What is the transpose of $[1, 2, 3]$?
4. What must each diagonal entry of a real skew-symmetric matrix be?
5. Can a 2×3 matrix be multiplied by a 3×4 matrix? State the product's order.

B. Short answers / practice - 2 marks each

6. For $A = [1, 2; 3, 4]$ and $B = [2, -1; 0, 5]$, find $A + B$ and $2A - B$.
7. Find x and y if $[x + 1, 2; 3, y - 1] = [4, 2; 3, 5]$.
8. For $A = [1, 2; 0, 1]$ and $B = [2, 0; 1, 3]$, compute AB and BA .
9. Write $A = [2, 3; 3, 5]$ as the sum of its symmetric and skew-symmetric parts.
10. Verify that $B = [1, -2; 0, 1]$ is the inverse of $A = [1, 2; 0, 1]$.
11. Find transpose of $[1, 3; 2, 4]$.
12. Can a 2×3 matrix be squared?
13. Find $3[1, -2; 0, 4]$.
14. If $A = [2, 0; 0, 3]$, find A^2 .
15. Is matrix multiplication always commutative?
16. Find x for symmetry of $[1, x; 5, 2]$.
17. Find x for skew-symmetry of $[0, x; -3, 0]$.
18. Solve $X + [1, 0; 0, 1] = [3, 2; 4, 5]$.
19. Find inverse of $[2, 0; 0, 5]$.
20. Prove that $A + A^T$ is symmetric for any square matrix A .

C. Short answers - 3 marks each

21. For $A = [1, 2; 3, 4]$, verify $(A^2)^T = (A^T)^2$ by calculation.

22. Find x such that $A = [2, x; 3, 1]$ satisfies $A^2 - 3A - 10I = 0$.

23. For $A = [1, 1; 0, 1]$, find A^2, A^3 and A^4 . Identify a pattern and verify it for the next power.

24. Give two non-zero 2×2 matrices whose product is zero. Explain why cancellation in matrix multiplication is not generally valid.

25. Solve $2X + A = B$ for $A = [1, 2; 3, 4]$ and $B = [5, 0; 1, 8]$. Verify your result.

D. Long answers - 5 marks each

26. Prove that every real square matrix is uniquely expressible as the sum of a symmetric and a skew-symmetric matrix. Illustrate with $A = [1, 4; -2, 3]$.
27. Prove uniqueness of the inverse of an invertible matrix. Then verify the inverse-product rule for $A = [1, 1; 0, 1]$ and $B = [2, 0; 0, 3]$.
28. For $A = [2, 1; 1, 2]$, verify $A^2 - 4A + 3I = 0$. Hence find A^{-1} and A^3 without direct repeated multiplication.
29. A shop sells notebooks and pens. Daily quantities on two days are rows of $Q = [10, 20; 15, 12]$, with prices ₹40 and ₹10. Use matrix multiplication to find each day's revenue, then find revenues if both prices rise by 10%.
30. Let $A = [0, 1; -1, 0]$. Find A^2, A^3 and A^4 , deduce A^{10} and A^{-1} , and verify that A is skew-symmetric but A^2 is symmetric.

ANSWER KEY - Numerical / short answers

- 1) 3×2 . 3) $[1, 2; 3]$. 4) Zero. 5) Yes; 2×4 . 6) $[3, 1; 3, 9]; [0, 5; 6, 3]$. 7) $x=3, y=6$. 8) $AB=[4, 6; 1, 3]; BA=[2, 4; 1, 5]$. 9) A +zero matrix. 10) $AB=BA=I$. 11) $[1, 2; 3, 4]$. 12) No. 13) $[3, -6; 0, 12]$. 14) $[4, 0; 0, 9]$. 15) No. 16) $x=5$. 17) $x=3$. 18) $X=[2, 2; 4, 4]$. 19) $[1/2, 0; 0, 1/5]$. 21) Both $[7, 15; 10, 22]$. 22) $x=4$. 23) $A^n=[1, n; 0, 1]$. 24) Example $A=[1, 0; 0, 0], B=[0, 0; 0, 1]$. 25) $X=[2, -1; -1, 2]$. 26) $S=[1, 1; 1, 3]; K=[0, 3; -3, 0]$. 27) $(AB)^{-1}=[1/2, -1/2; 0, 1/3]$. 28) $A^{-1}=(1/3)[2, -1; -1, 2]; A^3=[14, 13; 13, 14]$. 29) ₹600, ₹720; new ₹660, ₹792. 30) $A^2=-I, A^3=-A, A^4=I, A^{10}=-I, A^{-1}=-A$.