



Attempt all questions before checking the answer key. Use principal values. The notation $\sin^{-1}x$ means inverse sine, not $1/\sin x$.

A. Very short answers - 1 mark each

1. State the domain and principal range of $\sin^{-1}x$.
2. State the principal range of $\cos^{-1}x$.
3. Evaluate $\tan^{-1}(-1)$.
4. Evaluate $\sin^{-1}(\sin 5\pi/6)$.
5. State the domain of $\sec^{-1}x$ in the real number system.

B. Short answers / practice - 2 marks each

6. Evaluate $\cos^{-1}(\cos 7\pi/6)$.
7. Find $\sin[\cos^{-1}(3/5)]$.
8. Find $\cos[\sin^{-1}(-4/5)]$.
9. Find the domain of $\cos^{-1}(2x - 1)$.
10. Sketch $y = \sin^{-1}x$, marking endpoints and intercept.
11. Evaluate $\sin^{-1}(-1/2)$.
12. Evaluate $\cos^{-1}(-1/2)$.
13. Evaluate $\tan^{-1}\sqrt{3}$.
14. Find domain of $\sin^{-1}(3x)$.
15. Find $\cos(\tan^{-1}2)$.
16. Evaluate $\sin^{-1}1 + \cos^{-1}1$.
17. Is $\sin^{-1}(\sin x) = x$ for every real x ?
18. Find $\tan(\cos^{-1}(4/5))$.
19. Solve $\tan^{-1}x = \pi/4$.
20. Find principal value of $\sin^{-1}(\sin 4\pi/3)$.

C. Short answers - 3 marks each

21. Evaluate $\tan[\sin^{-1}(3/5)]$ and explain the sign using the principal range.
22. Find the domain of $\sin^{-1}[(x - 1)/2]$ and its range as x varies over that domain.

23. Simplify $\sin^{-1}(\sin x)$ for $\pi/2 \leq x \leq 3\pi/2$, giving the required pieces.

24. Evaluate $\cos[2 \sin^{-1}(1/3)]$ without approximations.

25. Solve $\cos^{-1}x = 2 \sin^{-1}x$ and check that the answer belongs to the domain.

D. Long answers - 5 marks each

26. Explain why sine must be restricted to define $\sin^{-1}$. Sketch $y = \sin x$ on its principal branch and $y = \sin^{-1}x$, and discuss their reflection relation.

27. Obtain the domain, range and endpoint values of $f(x) = \cos^{-1}[(3 - 2x)/5]$. Explain how you found the interval for x .

28. For $\theta = \tan^{-1}(3/4)$, find all six trigonometric ratios of θ , then determine the principal value of $\cos^{-1}(\cos 2\theta)$.

29. Solve $\sin^{-1}(2x) = \cos^{-1}x$. State the common domain, justify any squaring and check the solution in the original equation.

30. Sketch $y = \cos^{-1}x$ and $y = \tan^{-1}x$. Compare domains, ranges, monotonicity and symmetry properties, including the identity relating $\cos^{-1}(-x)$ and $\cos^{-1}x$.

ANSWER KEY - Numerical / short answers

- 1) Domain $[-1, 1]$; range $[-\pi/2, \pi/2]$. 2) $[0, \pi]$. 3) $-\pi/4$.
 4) $\pi/6$. 5) $(-\infty, -1] \cup [1, \infty)$. 6) $5\pi/6$. 7) $4/5$. 8) $3/5$.
 9) $[0, 1]$. 11) $-\pi/6$. 12) $2\pi/3$. 13) $\pi/3$. 14) $[-1/3, 1/3]$. 15) $1/\sqrt{5}$. 16) $\pi/2$. 17) No; only on the principal sine interval. 18) $3/4$. 19) $x=1$. 20) $-\pi/3$.
 21) $3/4$. 22) Domain $[-1, 3]$; range $[-\pi/2, \pi/2]$. 23) $\pi - x$ throughout the stated interval. 24) $7/9$. 25) $x=1/2$. 27) Domain $[-1, 4]$; range $[0, \pi]$; $f(-1)=0$, $f(4)=\pi$. 28) $\sin=3/5$, $\cos=4/5$, $\tan=3/4$, $\cot=4/3$, $\sec=5/4$, $\text{cosec}=5/3$; principal value $=2\tan^{-1}(3/4)$. 29) $x=1/\sqrt{5}$; common domain $[-1/2, 1/2]$.