



Attempt all questions before checking the answer key.
State the domain and codomain when testing a function.
Justify each relation property separately.

A. Very short answers - 1 mark each

1. Define a reflexive relation on a set A.
2. Define a symmetric relation.
3. Define a transitive relation.
4. What is a one-one function?
5. Can a function be onto if its range is a proper subset of its codomain? Explain.

B. Short answers / practice - 2 marks each

6. On $A = \{1, 2, 3\}$, test $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$ for reflexivity and symmetry.
7. Is $f: R \rightarrow R, f(x) = x^2$ one-one? Is it onto? Give reasons.
8. Show that $f: R \rightarrow R, f(x) = 3x - 2$ is one-one.
9. Determine whether $f: N \rightarrow N, f(n) = n + 1$ is one-one and onto.
10. On positive integers, test the relation aRb if a divides b for symmetry, giving a counterexample where needed.
11. Is the identity relation an equivalence relation?
12. Is $f: R \rightarrow R, f(x) = x^3$ bijective?
13. Find a preimage of 7 under $f(x) = 2x + 1$.
14. Is the empty relation on a non-empty set reflexive?
15. Test aRb iff $a \leq b$ on R for symmetry.
16. Find equivalence classes modulo 2 on Z .
17. Is $f: R \rightarrow R, f(x) = |x|$ onto?
18. Give a domain restriction making $f(x) = x^2$ one-one.
19. How many reflexive relations exist on a two-element set?
20. Prove the universal relation on a non-empty set is an equivalence relation.

C. Short answers - 3 marks each

21. Show that the relation on Z defined by aRb if $a - b$ is divisible by 5 is an equivalence relation.
22. Test the relation on R given by xRy if $|x - y| \leq 1$ for reflexivity, symmetry and transitivity.

23. Prove that $f: R \rightarrow (0, \infty), f(x) = e^x$ is one-one and onto.

24. Find whether $f: [0, \infty) \rightarrow [1, \infty), f(x) = x^2 + 1$ is bijective. Show how to find a preimage of any codomain element.

25. Let $A = \{1, 2, 3, 4\}$ and R be defined by aRb if a and b have the same parity. List the relation and identify its equivalence classes.

D. Long answers - 5 marks each

26. On Z , let aRb mean $a^2 = b^2$. Prove that R is an equivalence relation and describe the equivalence classes of 0, 2 and -3 .
27. Prove that $f: R - \{-2\} \rightarrow R - \{3\}, f(x) = (3x + 1)/(x + 2)$, is bijective. Explicitly solve $f(x) = y$ and check all exclusions.
28. On the set of all lines in a plane, test separately the relations "parallel or coincident" and "perpendicular" for reflexivity, symmetry and transitivity, with reasons.
29. For $A = \{1, 2, 3\}$, give one relation that is reflexive and symmetric but not transitive, and one that is reflexive and transitive but not symmetric. Verify each claim.
30. Show that $f: (-1, 1) \rightarrow R, f(x) = x/(1 - x^2)$, is one-one and onto without relying on a graph alone. You may use differentiation and limiting behaviour.

ANSWER KEY - Numerical / short answers

- 5) No. 6) Both reflexive and symmetric. 7) Neither one-one nor onto R . 9) One-one, not onto (1 has no preimage). 10) Not symmetric: 2 divides 4 but 4 does not divide 2. 11) Yes. 12) Yes. 13) 3. 14) No. 15) Not symmetric. 16) Even integers and odd integers. 17) No; negative numbers have no preimages. 18) $[0, \infty)$, for example. 19) 4. 22) Reflexive and symmetric, not transitive: $0R1, 1R2$ but not $0R2$. 24) Yes; $x = \sqrt{y-1}$. 25) $\{(1,1), (1,3), (3,1), (3,3), (2,2), (2,4), (4,2), (4,4)\}$; classes $\{1,3\}, \{2,4\}$. 26) $[0] = \{0\}; [2] = \{-2, 2\}; [-3] = \{-3, 3\}$. 27) $x = (1-2y)/(y-3), y \neq 3$. 28) Parallel/coincident: equivalence relation; perpendicular: symmetric only. 29) Example 1: diagonal pairs plus $(1,2), (2,1), (2,3), (3,2)$. Example 2: diagonal pairs plus $(1,2)$.