



Attempt questions first; check the separate key afterward. Use $GM = 4.0 \times 10^{14} \text{ m}^3/\text{s}^2$ and $R = 6.4 \times 10^6 \text{ m}$ for Earth. Neglect rotation and air resistance.

A. Very short answers - 1 mark each

1. State Newton's universal law of gravitation in scalar form. Define each symbol.
2. Write the SI unit and dimensional formula of G .
3. Why is gravitational potential energy negative when its value at infinity is taken as zero?
4. State Kepler's law of areas. Which conservation law explains it?
5. If Earth's mass doubles at unchanged radius, how does surface g change?
6. At what height above Earth is g one-fourth of its surface value?
7. Does the escape speed from a planet depend on the mass of the escaping body? Explain briefly.
8. Define gravitational potential at a point and state its SI unit.
9. What is the net gravitational field inside a uniform thin spherical shell?
10. Why does a planet move faster near perihelion than near aphelion?

B. Short answers / practice - 2 marks each

11. Two masses attract with force F . Find the new force if both masses double and their separation triples.
12. Compare G and g in terms of physical meaning, SI units and variation with location.
13. A planet has twice Earth's radius and the same mean density. Find the ratios of their masses and surface gravitational accelerations.
14. Find g at depth $R/4$ below Earth's surface. Assume uniform density and surface acceleration g_0 .
15. Two planets orbit the same star at radii r and $4r$. Find the ratios of their orbital speeds and periods.
16. A body weighs 72 N on Earth. Find its weight on a planet with half Earth's mass and one-third its radius.
17. A 1000 kg satellite moves in a circular orbit at radius $7.0 \times 10^6 \text{ m}$ from Earth's centre. Calculate its orbital speed.
18. Calculate the work done against gravity to move a 2 kg body slowly from Earth's surface to height R .
19. A planet's mass and radius are each four times Earth's. Compare their escape speeds and mean densities.
20. Find the gravitational potential at Earth's surface and at distance $2R$ from its centre. Take zero potential at infinity.

C. Application / reasoning - 3 marks each

21. Two point masses of 4 kg and 9 kg are 1 m apart. Locate the point between them where the net gravitational field is zero.
22. Obtain $g = GM/R^2$ and express g in terms of a spherical planet's mean density ρ and radius R .
23. A planet of mass $8M$ and radius $2R$ is compared with one of mass M and radius R . Find the ratios of surface g , escape speed and near-surface orbital period.
24. A satellite circles Earth at height $3R$. Find its orbital speed and period in terms of surface g and R .
25. At what height does g equal its value at depth $R/2$? Assume Earth has uniform density; use the exact altitude relation.
26. For a circular satellite orbit, show that $U = -2K$ and $E = -K$. Explain the significance of negative total energy.

27. State Kepler's first law. A planet's circular orbital radius is nine times Earth's orbital radius around the Sun. Find its period in Earth-years. Also compare their orbital speeds.

28. A body is launched vertically from Earth with speed half the escape speed. Using energy conservation, find its greatest height above the surface.

29. A planet's perihelion and aphelion distances are r and $5r$. Find the speed ratio at these points and explain the conservation law used.

30. A body has escape speed 12 km/s from a planet. If launched at 15 km/s, find its speed at infinity and explain the energy balance.

D. Extended practice - 5 marks each

31. Derive g at height h and obtain the approximation for $h \ll R$. Estimate the percentage decrease in g at height 32 km; take $R = 6400 \text{ km}$.

32. Assuming uniform Earth density, derive g at depth d . Sketch g versus distance from Earth's centre, inside and outside Earth, and mark its maximum.

33. Derive the escape-speed expression using conservation of energy. Calculate its value for Earth. State the assumptions and explain why continued propulsion changes this launch condition.

34. Derive the orbital speed and period of a satellite in a circular orbit. Hence establish Kepler's third law and explain why satellite mass does not enter the results.

35. Derive the gravitational potential energy of two point masses, choosing zero at infinite separation. Hence find the minimum energy needed to take a mass m from Earth's surface to infinity.

36. A satellite of mass m is moved from a circular orbit of radius $2R$ to one of radius $4R$. Obtain expressions for the changes in kinetic, potential and total energy. Interpret the required net energy input.

37. Derive gravitational potential $V = -GM/r$ outside a spherical mass. Sketch V against r for $r \geq R$. Hence obtain the radial gravitational field from V .

38. Show that a central gravitational force produces zero torque. Derive the relation between areal velocity and angular momentum, and hence establish Kepler's law of areas.

39. Derive escape speed in terms of a planet's mean density and radius. Two planets have equal mean density but radii R and $3R$; compare their escape speeds and surface g .

40. Derive the extra energy required to make a satellite escape from a circular orbit of radius r . Express it in terms of orbital kinetic energy; evaluate it for $m = 1000 \text{ kg}$ and $r = 2R$.

ANSWER KEY - Numerical / closed questions

- 2) $\text{N m}^2/\text{kg}^2$; $\text{M}^{-1}\text{L}^3\text{T}^{-2}$. 5) It doubles. 6) $h=R$. 7) No. 9) Zero. 11) $4F/9$. 13) Mass 8:1; g 2:1. 14) $3g_0/4$. 15) Speeds 2:1; periods 1:8. 16) 324 N. 17) 7.56 km/s. 18) $6.25 \times 10^7 \text{ J}$. 19) Escape speeds 1:1; densities 1:16. 20) $-6.25 \times 10^7 \text{ J/kg}$; $-3.125 \times 10^7 \text{ J/kg}$. 21) 0.40 m from 4 kg mass. 22) $g=4\pi G\rho R/3$. 23) g 2:1; escape 2:1; periods 1:1. 24) $v=\sqrt{(gR)/2}$; $T=16\pi\sqrt{(R/g)}$. 25) $(\sqrt{2}-1)R$. 27) 27 years; speeds 1:3. 28) $R/3$. 29) 5:1. 30) 9 km/s. 31) About 1% decrease (exact 0.9925%). 33) 11.18 km/s. 35) GMm/R . 36) $\Delta K = -GMm/(8R)$; $\Delta U = +GMm/(4R)$; $\Delta E = +GMm/(8R)$. 39) Escape 1:3; g 1:3. 40) $GMm/(2r) = \text{initial orbital } K$; $1.5625 \times 10^{10} \text{ J}$.