



Attempt questions first; check the separate key afterward. Take  $g = 10 \text{ m/s}^2$  where needed. Neglect air resistance unless stated.

### A. Very short answers - 1 mark each

1. Define the centre of mass of a two-particle system.
2. Can the centre of mass lie outside a body? Give an example.
3. Define torque and state its SI unit.
4. What is the condition for angular momentum conservation?
5. Why does moment of inertia depend on the chosen axis?
6. Define radius of gyration and relate it to moment of inertia.
7. State the two conditions for equilibrium of a rigid body.
8. For pure rotation about a fixed axis, do all particles have the same linear speed? Explain.

### B. Short answers / practice - 2 marks each

9. Masses 2 kg and 3 kg lie at  $x = 0$  and 5 m. Find their centre of mass.
10. Two particles of masses 1 and 2 kg have velocities  $6\mathbf{i}$  and  $-3\mathbf{i} \text{ m/s}$ . Find the centre-of-mass velocity.
11. A 10 N force acts perpendicular to a 0.4 m lever arm. Find torque; repeat when the angle is  $30^\circ$ .
12. For  $\mathbf{r} = 2\mathbf{i} + \mathbf{j} \text{ m}$  and  $\mathbf{F} = 3\mathbf{j} \text{ N}$ , find the torque vector about the origin.
13. A wheel starts from rest with angular acceleration  $2 \text{ rad/s}^2$ . Find angular speed and angle turned in 5 s.
14. A rim point 0.3 m from the axis has angular speed  $10 \text{ rad/s}$  and angular acceleration  $4 \text{ rad/s}^2$ . Find tangential and radial accelerations.
15. A ring and a disc each have mass 2 kg and radius 0.5 m. Find their moments of inertia about their symmetry axes; use  $I = MR^2$  and  $MR^2/2$ .
16. A body of mass 4 kg has moment of inertia  $1 \text{ kg m}^2$ . Find its radius of gyration.

### C. Application / reasoning - 3 marks each

17. Masses 1, 2 and 3 kg lie at (0, 0), (3, 0) and (0, 2) m. Find their centre of mass.
18. A 3 m uniform rod of weight 60 N is supported at both ends. A 30 N load is 1 m from its left end. Find both support reactions.
19. A couple consists of two opposite 15 N forces 0.20 m apart. Find its torque and explain why its resultant force is zero.

20. A uniform disc of mass 4 kg and radius 0.5 m rotates at  $8 \text{ rad/s}$ . Calculate its rotational energy and angular momentum; use  $I = MR^2/2$ .

21. A flywheel of  $I = 0.5 \text{ kg m}^2$  rotates steadily at  $40 \text{ rad/s}$  using a 100 W motor. Find resisting torque, rotational energy and angular momentum.

22. A 2 kg thin cylindrical shell of radius 0.25 m rotates on a fixed frictionless axis. A string pulls its rim with 8 N without slipping. Find angular acceleration and string acceleration; use  $I = MR^2$ .

23. A skater reduces her moment of inertia from 4 to 2  $\text{kg m}^2$  while rotating initially at  $3 \text{ rad/s}$ . Find final angular speed and energy change. Explain its source.

24. A 2 kg body at  $x = 0$  moves right at  $3 \text{ m/s}$ ; a 1 kg body at  $x = 6 \text{ m}$  is at rest. Find centre-of-mass position and velocity initially and after 2 s.

25. A wheel at  $20 \text{ rad/s}$  slows uniformly to rest through 50 rad. Find angular deceleration, stopping time and braking torque if  $I = 0.2 \text{ kg m}^2$ .

### D. Extended practice - 5 marks each

26. Derive the centre-of-mass position and velocity for two particles. Show that total momentum equals total mass times centre-of-mass velocity.
27. Derive  $\tau = dL/dt$  for a particle about a fixed origin. State and explain angular momentum conservation with two applications.
28. Derive  $K = I\omega^2/2$  for a rigid body rotating about a fixed axis. Obtain rotational work and power relations and compare with translational motion.
29. Derive the three equations of constant-angular-acceleration motion. A wheel speeds up from 2 to  $12 \text{ rad/s}$  in 5 s; find acceleration, angle and work if  $I = 0.4 \text{ kg m}^2$ .
30. Explain translational and rotational equilibrium. A uniform 4 m beam of weight 100 N has end supports and a 200 N load 1 m from its left end. Draw its force diagram and find reactions.

### ANSWER KEY - Numerical / closed questions

- 2) Yes; centre of a ring. 8) No;  $v = \omega r$  depends on distance from axis. 9) 3 m. 10) Zero. 11) 4 N m; 2 N m. 12)  $6\mathbf{k} \text{ N m}$ . 13)  $10 \text{ rad/s}$ ; 25 rad. 14)  $1.2 \text{ m/s}^2$  tangential;  $30 \text{ m/s}^2$  radial. 15) Ring 0.50; disc  $0.25 \text{ kg m}^2$ . 16) 0.50 m. 17) (1, 1) m. 18) Left 50 N; right 40 N. 19) 3 N m. 20) 16 J;  $4 \text{ kg m}^2/\text{s}$ . 21) 2.5 N m; 400 J;  $20 \text{ kg m}^2/\text{s}$ . 22)  $16 \text{ rad/s}^2$ ;  $4 \text{ m/s}^2$ . 23) 6 rad/s; energy increases 18 J due to internal muscular work. 24) Initially  $x_{cm} = 2 \text{ m}$ ; after 2 s  $x_{cm} = 6 \text{ m}$ ;  $v_{cm} = 2 \text{ m/s}$ . 25)  $\alpha = -4 \text{ rad/s}^2$ ;  $t = 5 \text{ s}$ ; torque  $= -0.8 \text{ N m}$ . 29)  $2 \text{ rad/s}^2$ ; 35 rad; 28 J. 30) Left 250 N; right 50 N.