



Attempt all questions before checking the answer key. Use radians for trigonometric limits. Show algebraic simplification before substituting at indeterminate forms.

A. Very short answers - 1 mark each

1. Find $\lim_{x \rightarrow 2} (3x^2 - x + 1)$.
2. State $\lim_{x \rightarrow 0} \sin x/x$.
3. Find $\lim_{x \rightarrow 0} (1 - \cos x)$.
4. What does $f'(a)$ represent geometrically?
5. Find the derivative of the constant function $f(x) = 7$.

B. Short answers / practice - 2 marks each

6. Evaluate $\lim_{x \rightarrow 3} (x^2 - 9)/(x - 3)$.
7. Evaluate $\lim_{x \rightarrow 0} \sin 5x/x$.
8. Evaluate $\lim_{x \rightarrow 0} [\sqrt{1+x} - 1]/x$.
9. Differentiate $x^3 - 4x^2 + 5x - 2$.
10. Differentiate $x \sin x$ using the product rule.
11. Evaluate $\lim_{x \rightarrow 1} (x^2 + 2x)$.
12. Evaluate $\lim_{x \rightarrow 0} \sin 7x/(2x)$.
13. Differentiate $5x^4 - 3x$.
14. Find derivative of $\cos x$.
15. Evaluate $\lim_{x \rightarrow 4} (\sqrt{x} - 2)/(x - 4)$.
16. Find derivative of $x^2 \sin x$.
17. Find slope of $y = x^3$ at $x = 2$.
18. Evaluate $\lim_{x \rightarrow 0} \tan x/x$.
19. Find derivative of $1/x$ using quotient rule.
20. Is $\lim_{x \rightarrow 0} |x|/x$ defined?

C. Short answers - 3 marks each

21. Evaluate $\lim_{x \rightarrow 2} (x^3 - 8)/(x^2 - 4)$.

22. Evaluate $\lim_{x \rightarrow 0} (1 - \cos 2x)/x^2$ using standard trigonometric limits.

23. Evaluate $\lim_{x \rightarrow 0} (e^x - 1)/x$ and $\lim_{x \rightarrow 0} \ln(1 + x)/x$, stating the standard limits used.

24. Differentiate $(x^2 + 1)/(x - 1)$ and state its domain.

25. Differentiate $\cos^2 x/(1 + \sin x)$ by first simplifying it; state where the original function is defined.

D. Long answers - 5 marks each

26. Using first principles, derive the derivative of x^2 and find the slope of its tangent at $x = 3$. Explain the limiting secant-line interpretation.

27. Using first principles and standard limits, derive the derivative of $\sin x$. Explain the need to measure x in radians.

28. Derive the product rule from the definition of derivative. Use it to differentiate $(x^2 + 1) \cos x$ without using a chain rule.

29. For a particle with position $s(t) = t^3 - 6t^2 + 9t$ metres, find its velocity, times of rest in $[0, 4]$, and the direction of motion in each interval.

30. Evaluate $\lim_{x \rightarrow 0} [\sin 3x + \sin 5x]/[\sin 2x + \sin 4x]$ and $\lim_{x \rightarrow 1} (x^4 - 1)/(x^3 - 1)$. Explain why direct substitution alone is insufficient.

ANSWER KEY - Numerical / short answers

1) 11. 2) 1. 3) 0. 5) 0. 6) 6. 7) 5. 8) $1/2$. 9) $3x^2 - 8x + 5$. 10) $\sin x + x \cos x$. 11) 3. 12) $7/2$. 13) $20x^3 - 3$. 14) $-\sin x$. 15) $1/4$. 16) $2x \sin x + x^2 \cos x$. 17) 12. 18) 1. 19) $-1/x^2, x \neq 0$. 20) No; one-sided limits are -1 and $+1$. 21) 3. 22) 2. 23) 1; 1. 24) $(x^2 - 2x - 1)/(x - 1)^2; x \neq 1$. 25) $-\cos x$; original requires $\sin x \neq -1$. 26) $2x$; slope 6. 27) $\cos x$. 28) $2x \cos x - (x^2 + 1) \sin x$. 29) $v = 3(t - 1)(t - 3)$; rest $t = 1, 3$; positive on $[0, 1)$, $(3, 4]$, negative on $(1, 3)$. 30) $4/3; 4/3$.