



Attempt all questions before checking the answer key.
State convergence conditions for infinite geometric series; distinguish a term from a partial sum.

A. Very short answers - 1 mark each

1. Find the fifth term of the GP 3, 6, 12, ...
2. Find the common ratio of 81, 27, 9, ...
3. Define arithmetic mean of two numbers.
4. Find the positive geometric mean of 4 and 25.
5. When does an infinite GP with first term a have sum $a/(1 - r)$?

B. Short answers / practice - 2 marks each

6. Find the n th term of the GP 5, -10, 20, ...
7. Find the sum of the first 6 terms of 2, 6, 18, ...
8. Insert three geometric means between 2 and 162, taking a positive common ratio.
9. Find the sum $12 + 6 + 3 + \dots$ to infinity.
10. If the arithmetic mean of two positive numbers is 13 and their geometric mean is 12, find the numbers.
11. Find sixth term of GP 2, 6, 18, ...
12. Find sum of first four terms of GP 1, 2, 4, ...
13. Find positive GM of 9 and 16.
14. Find sum to infinity of $8 + 2 + 1/2 + \dots$.
15. Can $1 + 2 + 4 + \dots$ have a finite sum to infinity?
16. Insert one positive geometric mean between 3 and 27.
17. Find common ratio if $a = 2$ and fourth term is 54.
18. Find AM of 7 and 13.
19. Find minimum of $x + 9/x$ for $x > 0$.
20. Find n if the n th term of 3, 9, 27, ... is 243.

C. Short answers - 3 marks each

21. The third term of a positive GP is 12 and its sixth term is 96. Find its first term, common ratio and sum of the first six terms.
22. Find n if $3 + 6 + 12 + \dots$ up to n terms has sum 381.

23. Express 0.272727... as a fraction using an infinite GP.

24. If a, b, c are consecutive terms of a GP, prove $b^2 = ac$. Does the converse hold for all real a, b, c ? Discuss the zero case.

25. A ball rebounds to $3/4$ of its previous height after each bounce. It is dropped from 16 m. Find total vertical distance travelled before coming to rest in the ideal model.

D. Long answers - 5 marks each

26. Derive the sum of n terms of a GP for $r \neq 1$. Obtain its infinite-sum formula with the necessary condition and discuss $r = 1$ separately.
27. Prove $AM \geq GM$ for two positive numbers, stating when equality holds. Use it to find the least value of $x + 16/x$ for $x > 0$.
28. The sum to infinity of a positive GP is 24 and the sum of its first two terms is 18. Find the first term, common ratio and smallest n for which the remaining sum is less than 1.
29. A machine worth ₹80,000 loses 10% of its value each year. Model its year-end values as a GP, find its value after 4 years and the total depreciation over those years.
30. Insert four positive geometric means between 3 and 96. Find their sum and product and verify the geometric-mean relation for each consecutive triple.

ANSWER KEY - Numerical / short answers

1) 48. 2) $1/3$. 3) $(a+b)/2$. 4) 10. 5) $|r| < 1$. 6) $5(-2)^{(n-1)}$. 7) 728. 8) 6, 18, 54. 9) 24. 10) 8 and 18. 11) 486. 12) 15. 13) 12. 14) $32/3$. 15) No. 16) 9. 17) 3. 18) 10. 19) 6 at $x=3$. 20) 5. 21) $a=3, r=2, S_6=189$. 22) 7. 23) $3/11$. 24) Converse fails without qualifications: $(0, 0, 1)$ satisfies $b^2=ac$ but is not a GP. 25) 112 m. 27) Minimum 8 at $x=4$. 28) $a=12, r=1/2$, smallest $n=5$. 29) ₹52488; depreciation ₹27512. 30) 6, 12, 24, 48; sum 90; product 82944.