



Attempt all questions before checking the answer key.
Use binomial expansions for positive integral powers.
Show coefficient calculations explicitly.

A. Very short answers - 1 mark each

1. State the binomial theorem for a positive integer exponent.
2. Write the coefficients in the expansion of $(a + b)^4$.
3. How many terms appear in the full expansion of $(a + b)^7$ before like terms are combined?
4. Find the constant term in $(2 + x)^3$ by direct expansion.

B. Short answers / practice - 2 marks each

5. Expand $(x + 2)^4$ completely.
6. Expand $(2x - 1)^4$ completely.
7. Use binomial expansion to calculate 102^3 .
8. Find the coefficient of x^2 by expanding $(1 + 3x)^4$.
9. Expand $(1+x)^3$.
10. Find coefficient of x in $(2+x)^4$.
11. Find constant term of $(3-x)^5$.
12. How many terms are in $(a+b)^{10}$?
13. Find sum of coefficients of $(1+2x)^4$.
14. Find coefficient of x^3 in $(1+x)^6$.
15. Expand $(x-1)^3$.
16. Evaluate 101^2 by binomial expansion.
17. Find coefficient of x^2 in $(2-x)^4$.
18. Show that $(a+b)^2 - (a-b)^2 = 4ab$.
19. Find $(1+x)^4 - (1-x)^4$.
20. Find coefficient of x^4 in $(1+2x)^5$.
21. Find sum of coefficients of $(2+x)^5$.
22. Does the binomial expansion of $(a+b)^n$ contain equal first and last coefficients?

C. Short answers - 3 marks each

23. Use expansions to simplify $(1 + x)^5 + (1 - x)^5$. Explain why odd powers cancel.
24. Use expansions to simplify $(a + b)^5 - (a - b)^5$ and factorise the result.
25. Evaluate 99^4 using the binomial theorem, keeping the arithmetic organised.
26. Expand $(x^2 + 1/x)^4$ for $x \neq 0$ and collect powers carefully.

D. Long answers - 5 marks each

27. Prove the binomial theorem using the combinatorial meaning of coefficients. Illustrate the proof by identifying all contributions to the x^2 term of $(1 + x)^4$.
28. Using binomial expansions, prove that the sum of even-position binomial coefficients equals the sum of odd-position coefficients for $n \geq 1$; find each sum for $n = 8$.
29. Expand $(1 + 2x)^5$ and $(1 - 2x)^5$. Hence find their sum and difference, and evaluate the resulting sum at $x = 1/2$.
30. Expand $(1 + x)^6$ and use it to estimate $(1.02)^6$ to four decimal places. Show the retained terms and check the size of the omitted terms.

ANSWER KEY - Numerical / short answers

- 2) 1,4,6,4,1. 3) 8. 4) 8. 5) $x^4 + 8x^3 + 24x^2 + 32x + 16$.
 6) $16x^4 - 32x^3 + 24x^2 - 8x + 1$. 7) 1061208. 8) 54. 9)
 $1 + 3x + 3x^2 + x^3$. 10) 32. 11) 243. 12) 11 before
 possible cancellation. 13) 81. 14) 20. 15)
 $x^3 - 3x^2 + 3x - 1$. 16) 10201. 17) 24. 19) $8x + 8x^3$.
 20) 80. 21) 243. 22) Yes; both are 1. 23)
 $2 + 20x^2 + 10x^4$. 24) $10a^4b + 20a^2b^3 + 2b^5 =$
 $2b(5a^4 + 10a^2b^2 + b^4)$. 25) 96059601. 26)
 $x^8 + 4x^5 + 6x^2 + 4/x + 1/x^4$. 27) Coefficient of x^2 is 6. 28)
 Each sum $= 2^{n-1}$; for $n=8, 128$. 29) Sum
 $2 + 80x^2 + 160x^4$; difference $20x + 160x^3 + 64x^5$; sum at $1/2$
 is 32. 30) 1.1262.