



Attempt all questions before checking the answer key. Let $i^2 = -1$. This sheet emphasises complex-number algebra and the Argand plane.

A. Very short answers - 1 mark each

1. Evaluate i^{27} .
2. Write the real and imaginary parts of $4 - 7i$.
3. Find the complex conjugate of $-3 + 2i$.
4. Find the modulus of $5 - 12i$.
5. Represent $-2 + 3i$ by coordinates in the Argand plane.

B. Short answers / practice - 2 marks each

6. Simplify $(3 + 2i) + (1 - 5i)$ and $(3 + 2i)(1 - 5i)$.
7. Express $(2 + 3i)/(1 - i)$ in the form $a + ib$.
8. Find the multiplicative inverse of $3 - 4i$.
9. Find real x and y if $(x + iy) + (2 - 3i) = 5 + 4i$.
10. Prove $z + \bar{z}$ is real and $z - \bar{z}$ is purely imaginary for any complex z .
11. Evaluate i^{40} .
12. Find $|-8+6i|$.
13. Simplify $(1+i)^2$.
14. Find z if $z+(2-i)=5+3i$.
15. Can a non-zero complex number have zero modulus?
16. Find $(2+i)(2-i)$.
17. Express $1/i$ in $a+ib$ form.
18. Find real x, y if $2x+iy=6-5i$.
19. Represent $|z|=2$ geometrically.
20. Show that z is real if and only if z equals its conjugate.

C. Short answers - 3 marks each

21. Simplify $[(1 + i)/(1 - i)]^6$ and explain the periodicity used.
22. Find real x and y if $(x + iy)(2 - i) = 7 + 4i$.
23. If $z = 2 - 3i$, find z^2 , $\bar{z}^2$ and $|z^2|$. Verify $|z^2| = |z|^2$.
24. Show that $(1 + i)^4 + (1 - i)^4$ is real, and evaluate it.
25. Describe and sketch the locus $|z - 2| = 3$ in the Argand plane.

D. Long answers - 5 marks each

26. For $z_1 = 1 + 2i$ and $z_2 = 3 - i$, compute $z_1 + z_2$, $z_1 z_2$ and z_1/z_2 . Verify the conjugate rule for each operation.
27. Prove $z \bar{z} = |z|^2$ and $|z_1 z_2| = |z_1| |z_2|$. Deduce the modulus of a quotient when the denominator is non-zero.
28. Find all real x and y for which $(x + iy)^2 = 3 + 4i$. Represent the resulting complex numbers in the Argand plane and find their moduli.
29. If $z = x + iy$ and $|z - 1| = |z + 1|$, derive the locus equation. Sketch it and interpret it geometrically.
30. Let $z = (a + i)/(a - i)$, $a \in \mathbb{R}$. Express z in Cartesian form, prove $|z| = 1$, and find all a for which z is purely imaginary.

ANSWER KEY - Numerical / short answers

- 1) $-i$. 2) 4 and -7 . 3) $-3-2i$. 4) 13. 5) $(-2,3)$. 6) $4-3i$; $13-13i$. 7) $-1/2+(5/2)i$. 8) $(3+4i)/25$. 9) $x=3$, $y=7$. 10) 1. 11) 10. 12) $2i$. 13) $3+4i$. 14) No. 15) 5. 16) $-i$. 17) $x=3$, $y=-5$. 18) Circle centred at origin, radius 2. 19) -1 . 20) $x=2$, $y=3$. 21) $-5-12i$; $-5+12i$; 13. 22) -8 . 23) Circle centre $(2,0)$, radius 3. 24) $4+i$; $5+5i$; $(1+7i)/10$. 25) $(x,y)=(2,1)$, $(-2,-1)$; modulus $\sqrt{5}$. 26) $x=0$, the imaginary axis. 27) $[(a^2-1)+2ai]/(a^2+1)$; $a=\pm 1$ for purely imaginary z .