



Attempt all questions before checking the answer key.  
Angles are in radians unless  $^\circ$  is shown. State restrictions wherever denominators occur.

### A. Very short answers - 1 mark each

1. Convert  $150^\circ$  into radians.
2. Convert  $7\pi/6$  radians into degrees.
3. State the domain and range of  $\tan x$ .
4. Find the sign of  $\sin x$  and  $\cos x$  when  $\pi < x < 3\pi/2$ .
5. Evaluate  $\sin(-\pi/6)$  and  $\cos(5\pi/3)$ .

### B. Short answers / practice - 2 marks each

6. A circle of radius 14 cm has an arc subtending  $45^\circ$  at its centre. Find arc length.
7. If  $\sin \theta = 3/5$  and  $\theta$  is in quadrant II, find  $\cos \theta$  and  $\tan \theta$ .
8. Use angle-addition formulae to find  $\sin 75^\circ$  and  $\cos 75^\circ$ .
9. Prove  $\cos(x + y) \cos(x - y) = \cos^2 x - \sin^2 y$ .
10. If acute A and B satisfy  $A + B = \pi/4$ , prove  $(1 + \tan A)(1 + \tan B) = 2$ .
11. Convert  $225^\circ$  into radians.
12. Find  $\cos(-\pi/3)$ .
13. Find  $\sin(7\pi/6)$ .
14. If  $\tan \theta = 3/4$  in quadrant III, find  $\sin \theta$ .
15. Evaluate  $\sin^2 75^\circ + \cos^2 75^\circ$ .
16. Find arc length for  $r=6$  cm and angle 2 radians.
17. Prove  $1 + \tan^2 x = \sec^2 x$  on its domain.
18. Express  $2\sin 3x \cos 2x$  as a sum.
19. Find  $\cos 2\theta$  if  $\sin \theta = 1/3$ .
20. Is  $\sin(x+y) = \sin x + \sin y$  for all real  $x, y$ ? Give a counterexample.

### C. Short answers - 3 marks each

21. If  $\cos \theta = -12/13$  and  $\theta$  is in quadrant III, find  $\sin 2\theta$ ,  $\cos 2\theta$  and  $\tan 2\theta$ .
22. Prove  $\sin 3x = 3 \sin x - 4 \sin^3 x$  using addition formulae.
23. Prove  $\cos 4x = 1 - 8 \sin^2 x \cos^2 x$ .
24. Express  $\sin 5x + \sin 3x$  and  $\cos 5x - \cos 3x$  as products.
25. Find exact values of  $\sin 15^\circ$ ,  $\cos 15^\circ$  and  $\tan 15^\circ$  without a calculator.

### D. Long answers - 5 marks each

26. Derive the formulae for  $\sin(x + y)$ ,  $\cos(x + y)$  and  $\tan(x + y)$ , explaining when the last expression is defined. Hence evaluate  $\tan 75^\circ$ .
27. Prove  $[\sin x + \sin 3x + \sin 5x]/[\cos x + \cos 3x + \cos 5x] = \tan 3x$  wherever both sides are defined.
28. If  $\tan A = 1/2$  and  $\tan B = 1/3$  for acute A, B, find  $\sin(A + B)$ ,  $\cos(A - B)$  and  $\tan 2A$ , with exact values.
29. Derive the double-angle and triple-angle formulae for cosine. Use them to express  $\cos^3 x$  and  $\cos^2 x$  in terms of multiple angles.
30. Sketch  $\sin x$ ,  $\cos x$  and  $\tan x$  over one suitable full period each. Mark zeros, extrema/asymptotes, periods and ranges, and explain their parity.

### ANSWER KEY - Numerical / short answers

- 1)  $5\pi/6$ . 2)  $210^\circ$ . 3) Domain  $\mathbb{R} - \{\pi/2 + n\pi\}$ ; range  $\mathbb{R}$ .  
4) Both negative. 5)  $-1/2$ ;  $1/2$ . 6)  $7\pi/2$  cm. 7)  $-4/5$ ;  $-3/4$ . 8)  $\sin 75^\circ = (\sqrt{6} + \sqrt{2})/4$ ;  $\cos 75^\circ = (\sqrt{6} - \sqrt{2})/4$ . 11)  $5\pi/4$ . 12)  $1/2$ . 13)  $-1/2$ . 14)  $-3/5$ . 15) 1. 16) 12 cm. 18)  $\sin 5x + \sin x$ . 19)  $7/9$ . 20) No;  $x=y=\pi/2$  gives  $0 \neq 2$ . 21)  $120/169$ ;  $119/169$ ;  $120/119$ . 24)  $2\sin 4x \cos x$ ;  $-2\sin 4x \sin x$ . 25)  $(\sqrt{6} - \sqrt{2})/4$ ;  $(\sqrt{6} + \sqrt{2})/4$ ;  $2 - \sqrt{3}$ . 26)  $\tan 75^\circ = 2 + \sqrt{3}$ . 28)  $1/\sqrt{2}$ ;  $7/\sqrt{50}$ ;  $4/3$ . 29)  $\cos^2 x = (1 + \cos 2x)/2$ ;  $\cos^3 x = (\cos 3x + 3\cos x)/4$ .